Kasdi Merbah University of Ouargla **Academic Year 2024 – 2025**
Faculty of Mathematics and Material Sciences **L1 (S1) Mathematics**

indefinite integrals and definite integrals (Serie 01)

<u>Exercise 1 :</u> Give the primitive functions of the following functions :

$$f_1 : x \longmapsto a, \quad f_2 : x \longmapsto x, \quad f_3 : x \longmapsto \frac{1}{x}, \quad f_4 : x \longmapsto \cos x, \quad f_5 : x \longmapsto \sin x$$

$$f_6 : x \longmapsto \cosh x, \quad f_7 : x \longmapsto \sinh x, \quad f_8 : x \longmapsto e^{2x}, \quad f_9 : x \longmapsto x^2, \quad f_{10} : x \longmapsto \frac{1}{\cos^2(x)}$$

<u>Exercise 2 :</u> Prove the validity of the following integrals :

$$1) \int \sqrt[n]{x^m} dx = \frac{n}{m+n} x^{\frac{m+n}{n}} + c_1, \quad 2) \int \left(\sqrt{x}+1\right)\left(x-\sqrt{x}+1\right) dx = \frac{2}{5}\left(\sqrt{x}\right)^5 + x + c_2,$$

$$3) \int \frac{\sqrt{x}+x^3e^x+x^2}{x^3} dx = \frac{-2}{3x\sqrt{x}} + e^x + \ln(x) + c_3, \quad 4) \int \frac{1+\cos^2(x)}{1+\cos(2x)} dx = \frac{1}{2}\tan x + \frac{1}{2}c_4 x + c_5,$$

$$5) \int \arctan(x)\, dx = x \arctan(x) - \frac{1}{2}\ln\left(x^2+1\right) + c_6,$$

where c_1, c_2, c_3, c_4, c_5 and c_6 are real constants.

<u>Exercise 3 :</u> 1) Calculate the following integrals :

$$I_1 = \int_0^1 \left(x^2+x\right) dx, \quad I_2 = \int_1^2 \left(e^x + \frac{1}{x}\right) dx, \quad I_3 = \int_0^{\frac{\pi}{2}} (\cos x + \sin x)\, dx, \quad I_4 = \int_0^1 \frac{1}{1+x^2} dx,$$

$$I_5 = \int_0^1 x\sqrt{x^2+1} dx, , \quad I_6 = \int_0^{\frac{\pi}{4}} \sin^3(x)\cos(x)\, dx, \quad I_7 = \int_0^{\frac{\pi}{2}} \frac{\cos(x)}{\sin x + \cos x} dx,$$

2) Using integration by parts, calculate the following integrals :

$$I_8 = \int_1^2 x^2 \ln(x)\, dx, , \quad I_9 = \int_0^1 \ln\left(1+x^2\right) dx, \quad I_{10} = \int_0^1 \arctan(x)\, dx.$$

<u>Exercise 4 :</u> Calculate the integrals of the following rational functions :

$$I_1 = \int_0^1 \frac{x^2+x}{x^2+2x+1} dx, \quad I_2 = \int_1^2 \frac{x+1}{x^2+x+1} dx, \quad I_3 = \int_2^3 \frac{x+2}{x(x-1)} dx,$$

$$I_4 = \int\limits_0^{\frac{\pi}{4}} \frac{1}{1+\tan^2(x)} dx, \quad I_5 = \int\limits_{-\frac{\pi}{2}}^{0} \frac{-\cos^2(x) + -1}{1+\cos x} dx, \quad I_6 = \int\limits_1^4 \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx,$$

$$I_7 = \int\limits_0^1 \frac{\sqrt{x}}{1+\sqrt{x}} dx, \quad I_8 = \int\limits_0^{\sqrt{2}} \frac{x}{x^4+4} dx.$$

Exercise 5 : We consider the integral $I_n = \int\limits_0^{\frac{\pi}{2}} x^n \sin(x)\, dx$, for every natural number n of $\mathbb{N}$.

1) Calculate I_0 and I_1.

2) Using integration by parts, find a recurring relation between I_n and I_{n+2}, then deduce I_2 and I_3.

Exercise 6 : 1) Calculate the following two integrals ($[\cdot]$ denotes the integer part) :

$$I_1 = \int\limits_{-1}^{4} [x]\, dx, \qquad I_2 = \int\limits_0^1 \left[3x^2\right] dx.$$

2) Prove that : $\lim_{n \longrightarrow \infty} \int\limits_0^{2\pi} \frac{\sin(nx)}{n^2+x^2} dx = 0.$

Exercise 7 : Using the Riemann sum, calculate the following integral : $\int\limits_0^1 x^2 dx.$

Reminder : $\sum\limits_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$

Exercise 8 : Using the Cauchy-Schwarz inequality, prove that for $b > a > 0$ then :

$$\int\limits_a^b \frac{1}{x} dx < \frac{b-a}{\sqrt{ab}}.$$

Exercise 9 : Let n be from $\mathbb{N}$ and $I_n = \int\limits_0^{\frac{\pi}{4}} \tan^n(x)\, dx.$

1- Prove that : $\forall n \in \mathbb{N};\ I_n > 0.$

2- Determine the relationship between I_n and I_{n+2}.

3- Prove that the sequence (I_n) is convergent, then calculate its limit.

4- Calculate (I_1) and then conclude (I_3).

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Exercise 3 : Calculate de definite integrals :

$$I_1 = \frac{5}{6}, \qquad I_2 = e^2 + \ln(2) - e, \qquad I_3 = 2, \qquad I_4 = \frac{\pi}{4},$$
$$I_5 = \frac{1}{3}\left(2\sqrt{2} - 1\right), \qquad I_6 = \frac{1}{16}, \qquad I_7 = \frac{\pi}{4}.$$

2) Using integration by parts, calculate the following integrals :

$$I_8 = \frac{8}{3}\ln(2) - \frac{7}{9}, \qquad I_9 = \ln(2) + \frac{\pi}{2} - 2, \qquad I_{10} = \frac{\pi}{4} - \frac{1}{2}\ln(2).$$

Exercise 4 : Calculate the integrals of the following rational functions :

$$I_1 = 1 - \ln(2), \qquad I_2 = \frac{1}{2}\ln(\frac{7}{3}) + \frac{1}{\sqrt{3}}\arctan(\frac{5}{\sqrt{3}}) - \frac{\pi}{3\sqrt{3}},$$
$$I_3 = 5\ln(2) - 2\ln(3), \qquad I_4 = \frac{1}{4} + \frac{\pi}{8}, \qquad I_5 = \frac{\pi}{2} + 1,$$

$$I_6 = \frac{4}{3}\left(3\sqrt{3} - 2\sqrt{2}\right), \qquad I_7 = 2\ln(2) - 1, \qquad I_8 = \frac{\pi}{16}.$$

Exercise 5 :
1) $I_0 = 1$ and $I_1 = 1$.
2) The reccuring relation between I_n and I_{n+2} is given by :

$$I_{n+2} = (n+2)\left(\frac{\pi}{2}\right)^{n+1} - (n+1)(n+2)I_n.$$

If $n = 0$ we get : $I_2 = \pi - 2$.
If $n = 1$ we get : $I_3 = \frac{3\pi^2}{4} - 6$.

Exercise 6 : 1) Calculate the following two integrals ($[\cdot]$ denotes the integer part) :

$$I_1 = \int_{-1}^{4} [x]\, dx = 5, \qquad I_2 = \int_{0}^{1} \left[3x^2\right] dx = \frac{2\sqrt{3} - \sqrt{2} - 1}{\sqrt{3}}.$$

2) Prove that : $\lim_{n \longrightarrow \infty} \int_0^{2\pi} \frac{\sin(nx)}{n^2+x^2} dx = 0$. (we use the squeeze theorem)

Exercise 7 : Using the Riemann sum, we get : $\int\limits_0^1 x^2 dx = \frac{1}{3}$.

Exercise 9 : Let n be from $\mathbb{N}$ and $I_n = \int\limits_0^{\frac{\pi}{4}} \tan^n(x)\, dx$.

1- Prove that : $\forall n \in \mathbb{N};\ I_n > 0$.
2- The reccuring relation between I_n and I_{n+2} is given by :

$$I_{n+2} + I_n = \frac{1}{n+1}.$$

3- The sequence (I_n) is convergent, and

$$\lim_{n \longrightarrow +\infty} I_n = 0.$$

4- If $n = 1$ we use integral, we get : $I_1 = \ln \frac{\sqrt{2}}{2}$.
If $n = 1$ we get : $I_3 = \frac{1}{2} - \ln \frac{\sqrt{2}}{2}$. .
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