

**Subject: Optimization 1**  
–Tutorial 01 –

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**Exercise 01: Rayleigh Quotient**

Consider the function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  defined by the relation:

$$f(x) = \frac{\langle Ax, x \rangle}{\langle x, x \rangle}$$

where  $A$  is a positive definite symmetric matrix of dimension  $n$ .

- Calculate  $DF(x)$  and  $\nabla f(x)$ .

**Exercise 02:**

Consider the function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  defined by the relation:

$$f(x) = \frac{1}{2} \langle Ax, x \rangle + \langle b, x \rangle$$

where  $A$  is a positive definite symmetric matrix of dimension  $n$ , and  $b \in \mathbb{R}^n$ .

- Calculate  $DF(x)$  and  $\nabla f(x)$ .

**Exercise 03:**

Consider the function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  defined by the relation:

$$f(x) = \sin(\|x\|^2) = \sum_{i=1}^{i=n} x_i^2$$

- Show that the function is differentiable on  $\mathbb{R}^n$  and calculate its gradient.
- Show that the function  $f$  is twice differentiable ( $C^2$ ) and calculate its Hessian matrix.

**Exercise 04:**

Consider the Euclidean space  $\mathbb{R}^n$  equipped with the standard inner product. Let  $y$  be a fixed non-zero vector, and define the function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  as follows:

$$\forall x \in \mathbb{R}^n : f(x) = \langle x, y \rangle e^{-\|x\|},$$

where  $\langle \cdot, \cdot \rangle$  denotes the dot product.

- Calculate the partial derivatives of  $f$  and deduce its gradient.