FMSM-Mathematics | University of Ouargla
Bachelor's Degree L03 - Semester 1 | 2024-2025

Module:Optimization 1
–Tutorial 04 –

Exercise 01: Rayleigh Quotient

Consider the function $f : \mathbb{R}^n \to \mathbb{R}$ defined by the relation:

$$f(x) = \frac{\langle Ax, x\rangle}{\langle x, x\rangle}$$

where A is a positive definite symmetric matrix of dimension n.

- Calculate $DF(x)$ and $\nabla f(x)$.
- Determine the points for which $\nabla f(x) = 0$.

Exercise 02:

Exercise 2: Consider the following function from $\mathbb{R}^2$ to $\mathbb{R}$:

$$f(x, y) = 2x^3 + 6xy - 3y^2 + 2$$

- Determine the critical points of f.
- For each critical point obtained, determine if f has a local minimum or a local maximum at that point, or if it is a saddle point.
- Determine the global maxima and minima of f.

Exercise 03:

Consider the function $f : \mathbb{R}^n \to \mathbb{R}$ defined by the relation:

$$f(x) = \sin(\|x\|^2) = \sin \sum_{i=1}^{i=n} (x_i^2)$$

- Show that the function is differentiable on $\mathbb{R}^n$ and Calculate $DF(x)$ and $\nabla F(x)$. Determine the points for which $\nabla F(x) = 0$.
- Show that the function f is twice differentiable (C^2) and calculate its Hessian matrix $HF(x)$.What is the nature of the extrema?

Exercise 04:

Consider the Euclidean space $\mathbb{R}^n$ equipped with the standard inner product. Let y be a fixed non-zero vector, and define the function $f : \mathbb{R}^n \to \mathbb{R}$ as follows:

$$\forall x \in \mathrm{R} : f(x) = \langle x, y\rangle \, e^{-\|x\|},$$

where $\langle \cdot, \cdot \rangle$ denotes the dot product.

- Calculate the partial derivatives of f and deduce its gradient.
- Determine the critical points of f.