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Mathematics Department 2024-2025 academic year

Exercise set 5 - Topology

Exercise 1: Let E be a topological space, A a connected subset of E, and B a subset of E such that:

$$A \subset B \subset \overline{A}.$$

Show that B is connexe.

Exercise 2: Let E be a topological space, A and B two closed sets in E such that $A \cup B$ and $A \cap B$ are connected.

(a) Show that A and B are connected.

(b) Give an example on $\mathbb{R}$ to show that the closed condition is necessary.

Exercise 3: Let E and F be two topological spaces and $G = E \times F$ their product. Show that G is connected if and only if E and F are connected.

Exercise 4: Let A and B be two connected sets in a topological space E such that $A \cap \overline{B} \neq \emptyset$ or $\overline{A} \cap B \neq \emptyset$; show that $A \cup B$ is connected.

Exercise 5: Prove that if E has a finite number of connected components, then each connected component is both open and closed.

Exercise 6: Show that if A is a non-empty connected set in a topological space E that is both open and closed, then A is a connected component.

Exercise 7: Show that a space equipped with the coarse topology is path-connected.

Exercise 8: 1) Show that if E is a path-connected space, then E is connected.

2) Explain why any **convex** subset of $\mathbb{R}^n$ is **connected**.