University Kasdi Merbah-Ouargla

Department of mathematics

Abstract Algebra 01

2nd series of exercises: Sets and binary relations, equivalence and order (Tutorials) 2024/2025

Exercise 01

1/Describe the following sets using Roster method:

$$A = \{\frac{1}{n} \mid n \in \{3,4,5,6\}\} \qquad B = \{x \in \mathbb{Z} \mid x = x+1\}$$

2/ Let A and B be sets. Show that

$$a)\ A \cap (B \backslash A) = \varnothing \qquad b)\ A \cup (B \backslash A) = A \cup B \qquad c)\ \mathcal{P}(A) \subseteq \mathcal{P}(B) \iff A \subseteq B.$$

Exercise 02 : (De Morgan laws)

Show that:

a) $(A \cup B)^c = (A^c \cap B^c)$

b) $(A \cap B)^c = (A^c \cup B^c)$

Exercise 03:

Let E be a set and A,B and C powersets of E.

a) Give the definition of $A \triangle B$.

b) We assume that $A \triangle B = A \triangle C$, establish that $B \subset C$. (Check the two cases $x \in A$ and $x \notin A$).

Exercise 04:

Let $\mathcal{R}$ be the relation defined on $\mathbb{R}$ by:

$$x\mathcal{R}y \Leftrightarrow x^2 - y^2 = x - y$$

1) Show that $\mathcal{R}$ is an equivalence relation.

2) What are the equivalence classes of a given real x, as well as $cl(0)$.

Exercise 05:

Let E be a finite set, show if the following relation is reflexive, antisymmetric and transitive?

$$\forall A, B \in \mathcal{P}(E), A\mathcal{R}B \Leftrightarrow x \in A \cap B^c$$

Exercise 06:

Let $\mathcal{R}$ be the relation defined on $\mathbb{R}^2$ by:

$$(x, y)\mathcal{R}(x', y') \Leftrightarrow |x - x'| \leqslant y' - y$$

Show that $\mathcal{R}$ is an order relation.

Exercise 07:

Let $\mathcal{R}$ be a relation defined on $\mathbb{R}$ by : $\forall (x, y) \in \mathbb{R}^2,\ x\mathcal{R}y \Longleftrightarrow \cos^2(x) + \sin^2(y) = 1$.

1) Prove that $\mathcal{R}$ is an equivalence relation on $\mathbb{R}$.

2) Find the equivalence class of 0.

Exercise 08:

Let $\mathcal{R}$ be a relation defined on $\mathbb{N}$ by : $\forall n, m \in \mathbb{N},\ n\mathcal{R}m \Longleftrightarrow m$ divise n.

Prove that $\mathcal{R}$ is a partial order relation on $\mathbb{N}$.

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Abstract Algebra 01

2nd series of exercises: Functions (Tutorials) 2024/2025)

Exercise 01:

For $f : \mathbb{R} \to \mathbb{R},\ x \longmapsto x^2 + 1$, find each of the following :

1/ $f^{-1}([-1,1])$ and $f^{-1}([0,10])$

2/] $f([-1,1])$ and $f([1,3])$

Exercise 02:

Is the following functions injectives, surjectives, bijectives ?

1) $f : [0,+\infty[\longrightarrow [1,+\infty[$
$x \longrightarrow f(x) = 3x^2 + 4x + 1.$

2) $g : [-1,1] \longrightarrow \mathbb{R}$
$x \longrightarrow g(x) = \sqrt{1-x^2}.$

Exercise 03:

We consider the following function :

$$g :]-\infty, 0[\longrightarrow]1,+\infty[$$
$$x \longmapsto g(x) = 1 + \frac{1}{x^2}.$$

1. Show that g is bijective.
2. Find g^{-1}.

Exercise 04:

For $f : \mathbb{R} \to \mathbb{R},\ x \longmapsto x^2 + 1$, find each of the following :

1/ $f^{-1}([-1,1])$ and $f^{-1}([0,10])$

2/] $f([-1,1])$ and $f([1,3])$

Exercise 05:

We consider the following function :

$$g :]-\infty, 0[\longrightarrow]1, +\infty[$$
$$x \longmapsto g(x) = 1 + \frac{1}{x^2}.$$

1. Show that g is bijective.
2. Find g^{-1}.